

# Math Common Core State Standards and Long-Term Learning Targets

## High School Algebra 1

Traditional Pathway; see Appendix A of the CCS Standards for information on high school course design: [http://www.corestandards.org/assets/CCSSI\\_Mathematics\\_Appendix\\_A.pdf](http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf)

*Note: Students should be able to apply all mathematical skills in context (through a word problem, open-ended real-world problem, or contextual scenario) and abstractly (in plain number problems or what the standards term "mathematical problems"). For example, when students are asked to "write, solve, and interpret two-step equations" students should be able to solve equations such as  $3x + 2 = -5$ , and check for the validity of their solution as well as write equations from word problems.*

<b>Unit 1: Relationships between Quantities and Reasoning with Equations</b>	
<b>CCS Standards: Quantities</b>	<b>Long-Term Target(s)</b>
N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	I can choose, apply, and interpret the units for multi-step problems when using formulas, graphs, and other data displays.
N-Q.2. Define appropriate quantities for the purpose of descriptive modeling.	I can analyze data to determine significant patterns (units or scale) that can result in a mathematical model.  I can determine appropriate variables from data.  I can determine the appropriate units and scale to model data.
N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	I can record data to an appropriate level of accuracy when using different types of measuring devices (e.g. traditional ruler vs. electronic measuring device, stopwatch vs. clock).  I can calculate using an appropriate level of accuracy.
<b>CCS Standards: Seeing Structure in Expressions</b>	<b>Long-Term Target(s)</b>
A-SSE.1. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)^n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i>	I can interpret algebraic expressions that describe real-world scenarios. This means :  <ul style="list-style-type: none"> <li>• I can interpret the parts of an expression including the factors, coefficients, and terms.</li> <li>• I can use grouping strategies to interpret expressions.</li> </ul>

CCS Standards: Creating Equations	Long-Term Target(s)
A-CED.1. Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i>	I can write, solve, and interpret linear and simple exponential equations and inequalities.
A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	I can write and graph equations that represent relationships between two variables or quantities.
A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i>	I can represent constraints with linear equations, inequalities, and systems of equations or inequalities.  I can determine whether solutions are viable or non-viable options, given the constraints provided in a modeling context.
A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's law <math>V = IR</math> to highlight resistance <math>R</math>.</i>	I can solve formulas for a particular variable of interest.
CCS Standards: Reasoning with Equations and Inequalities	Long-Term Target(s)
A-REI.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	I can explain and justify each step for solving multi-step linear equations.
A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	I can solve multi-step linear equations in one variable including equations with coefficients represented by letters.  I can solve multi-step linear inequalities in one variable.

## Unit 2: Linear and Exponential Relationships

CCS Standards: Real Number System	Long-Term Target(s)
N.RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For example, we define <math>5^{1/3}</math> to be the cube root of 5 because we want <math>(5^{1/3})^3 = 5(1/3)^3</math> to hold, so <math>(5^{1/3})^3</math> must equal 5.</i>	I can describe the relationship between rational exponents and radicals.
N.RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	I can rewrite expressions that contain radicals and/or rational exponents using the properties of exponents.
CCS Standards: Reasoning with Equations and Inequalities	Long-Term Target(s)
A.REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.  A.REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	I can write, solve, interpret, and justify my solution method for systems of linear equations using multiple methods (linear combination, substitution, and graphing).
A.REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).  A.REI.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	I can describe and interpret the solution set of a system of equations graphically and relate that to the algebraic solution.

<p>A.REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p>	<p>I can describe and interpret the solutions to a system of linear inequalities graphically.</p>
<p><b>CCS Standards: Interpreting Functions</b></p>	<p><b>Long-Term Target(s)</b></p>
<p>F.IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain, then <math>f(x)</math> denotes the output of <math>f</math> corresponding to the input <math>x</math>. The graph of <math>f</math> is the graph of the equation <math>y = f(x)</math>.</p>	<p>I can determine if a relation is a function.</p> <p>I can represent a function using a graph, table, and equation and describe the relationship between each form using function notation.</p>
<p>F.IF.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p>	<p>I can evaluate a function using function notation and interpret the value in context.</p> <p>I can determine the domain and range of a function.</p>
<p>F.IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1</math>, <math>f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i></p>	<p>I can write a linear or exponential function from a sequence.</p>

<p>F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p> <p>F.IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i></p> <p>F.IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p>	<p>I can interpret the graphical representation of linear and exponential functions.</p> <p>This means:</p> <ul style="list-style-type: none"> <li>• I can identify and interpret an appropriate domain and range.</li> <li>• I can interpret key elements of the graph, including average rate of change, y-intercept, x-intercepts.</li> <li>• I can sketch a graph showing key features given a particular scenario or context.</li> </ul>
<p>F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p>	<p>I can graph linear, exponential, and quadratic functions that are expressed symbolically. This means:</p> <ul style="list-style-type: none"> <li>• I can show intercepts, maxima, and minima.</li> <li>• I can graph piecewise-defined functions, including step functions and absolute value functions.</li> </ul>
<p>F.IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>	<p>I can compare two functions that are each represented differently (graphs, tables, equations, verbal descriptions).</p>

CCS Standards: Building Functions	Long-Term Target(s)
<p>F.BF.1. Write a function that describes a relationship between two quantities.</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>b. Combine standard function types using arithmetic operations.</p> <p><i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></p>	<p>I can write, evaluate, graph, and interpret linear and exponential functions that model the relationship between two quantities.</p>
<p>F.BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</p>	<p>I can explain that sequences are functions and are sometimes defined recursively.</p>
<p>F.BF.3. Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>kf(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>	<p>I can determine the effect of a transformational constant on a linear function.</p>
CCS Standards: Linear, Quadratic, & Exponential Models	Long-Term Target(s)
<p>F.LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <p>a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.</p> <p>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</p> <p>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</p>	<p>I can analyze a given context to determine whether it can be modeled with a linear or an exponential function.</p>

F.LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	I can analyze an arithmetic or geometric sequence to determine a corresponding linear or exponential function.
F.LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	I can compare and draw conclusions about graphs and tables of linear and exponential functions.
F.LE.5. Interpret the parameters in a linear or exponential function in terms of a context.	I can interpret the parameters in linear and exponential function models, in terms of their contexts.
<b>Unit 3: Descriptive Statistics</b>	
<b>CCS Standards: Interpreting Categorical &amp; Quantitative Data</b>	<b>Long-Term Target(s)</b>
S.ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	I can represent data with dot plots, histograms, and box plots.
S.ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	I can compare the center (mean and median) and spread (interquartile range and standard deviation) of two or more data sets based on the shape of the data distribution.
S.ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	I can interpret differences in shape, center, and spread based on the context of the data set and determine possible effects of outliers on these measures.

<p>S.ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.</p> <p>S.ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p> <p>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. <i>Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.</i></p> <p>b. Informally assess the fit of a function by plotting and analyzing residuals.</p> <p>c. Fit a linear function for a scatter plot that suggests a linear association.</p>	<p>I can summarize, represent, and interpret categorical and quantitative data based on two variables (independent and dependent).</p> <p>Summarize means:</p> <ul style="list-style-type: none"> <li>• I can create a two-way frequency table.</li> <li>• I can interpret relative frequencies given the context of the data.</li> <li>• I can recognize possible associations and trends in the data.</li> </ul> <p>Represent means:</p> <ul style="list-style-type: none"> <li>• I can show two variable data on a scatter plot.</li> <li>• I can describe the relationship between the variables.</li> <li>• I can identify a function of best fit for the data set.</li> <li>• I can assess the fit of a function to a data set.</li> </ul>
<p>S.ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</p>	<p>I can interpret the slope and intercept of a linear model based on context.</p>
<p>S.ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.</p>	<p>I can compute and interpret the correlation coefficient of a linear fit.</p>
<p>S.ID.9. Distinguish between correlation and causation.</p>	<p>I can describe the difference between correlation and causation.</p>
<b>Unit 4: Expressions and Equations</b>	
<p><b>CCS Standards: Seeing Structure in Expressions</b></p>	<p><b>Long-Term Target(s)</b></p>
<p>A.SSE.1. Interpret expressions that represent a quantity in terms of its context.</p> <p>a. Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret <math>P(1+r)n</math> as the product of <math>P</math> and a factor not depending on <math>P</math>.</i></p>	<p>I can create and interpret quadratic and exponential algebraic expressions to describe real-world scenarios.</p>



<p>A.SSE.2. Use the structure of an expression to identify ways to rewrite it. <i>For example, see <math>x^4 - y^4</math> as <math>(x^2)^2 - (y^2)^2</math>, thus recognizing it as a difference of squares that can be factored as <math>(x^2 - y^2)(x^2 + y^2)</math>.</i></p>	<p>I can identify the structure of a quadratic expression in order to rewrite it.</p> <p>This means:</p> <ul style="list-style-type: none"> <li>• I can recognize the difference of squares.</li> <li>• I can recognize a quadratic perfect square trinomial.</li> </ul>
<p>A.SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <p>a. Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</p> <p>c. Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression <math>1.15t</math> can be rewritten as <math>(1.151/12)12t \approx 1.01212t</math> to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i></p>	<p>I can determine if rewriting an expression will reveal important properties of the expression.</p> <p>I can factor a quadratic expression in order to reveal its zeros.</p> <p>I can complete the square of a quadratic expression to reveal the maximum or minimum value of the function.</p> <p>I can use the properties of zero and 1 to produce an equivalent form of an expression.</p>
<p><b>CCS Standards: Arithmetic with Polynomials &amp; Rational Expressions</b></p>	<p><b>Long-Term Target(s)</b></p>
<p>A.APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p>	<p>I can identify a polynomial expression.</p> <p>I can add, subtract, and multiply polynomials.</p>
<p><b>CCS Standards: Creating Equations</b></p>	<p><b>Long-Term Target(s)</b></p>
<p>A.CED.1. Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A.CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p>	<p>I can write and interpret quadratic equations and inequalities mathematically and in context, graphically and algebraically.</p>

<p>A.CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. <i>For example, rearrange Ohm's Law <math>V = IR</math> to highlight resistance <math>R</math>.</i></p>	<p>I can rearrange a formula with squared exponents to highlight a particular quantity.</p>
<p><b>CCS Standards: Reasoning with Equations &amp; Inequalities</b></p>	<p><b>Long-Term Target(s)</b></p>
<p>A.REI.4. Solve quadratic equations in one variable.  a. Use the method of completing the square to transform any quadratic equation in <math>x</math> into an equation of the form <math>(x - p)^2 = q</math> that has the same solutions. Derive the quadratic formula from this form.  b. Solve quadratic equations by inspection (e.g., for <math>x^2 = 49</math>), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.</p>	<p>I can determine whether the solution of a quadratic equation will be real or complex.</p> <p>I can find real solutions to quadratic equations in one variable using multiple methods and justify my solution method.</p>
<p>A.REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. <i>For example, find the points of intersection between the line <math>y = -3x</math> and the circle <math>x^2 + y^2 = 3</math>.</i></p>	<p>I can solve a system of equations consisting of a linear equation and quadratic equation algebraically and graphically.</p>
<p><b>Unit 5: Quadratic Equations</b></p>	
<p><b>CCS Standards: Real Number System</b></p>	<p><b>Long-Term Target(s)</b></p>
<p>N.RN.3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</p>	<p>I can explain which operations are closed in the set of real numbers and its subsets of rational and irrational numbers.</p>

CCS Standards: Interpreting Functions	Long-Term Target(s)
<p>F.IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p>	<p>I can analyze a quadratic model based on a verbal description. This means:</p> <ul style="list-style-type: none"> <li>• I can sketch a reasonable graph of a quadratic function based on a verbal description.</li> <li>• I can identify the intercepts, intervals for which the function is increasing, decreasing, positive, or negative on a graph or table.</li> <li>• I can determine a local maximum or minimum.</li> <li>• I can find the line of symmetry.</li> </ul>
<p>F.IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.</i></p>	<p>I can determine the appropriate domain of a quadratic function given its real-world context.</p>
<p>F.IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</p>	<p>I can use a graph to describe how a quadratic function is changing (rate of change) over a given interval.</p> <p>I can estimate the rate of change over a given interval from a graph.</p>
<p>F.IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p>	<p>I can graph linear, exponential, and quadratic functions that are expressed symbolically. This means:</p> <ul style="list-style-type: none"> <li>• I can show intercepts, maxima, and minima.</li> <li>• I can graph piecewise-defined functions, including step functions and absolute value functions.</li> </ul>

<p>F.IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. <i>For example, identify percent rate of change in functions such as <math>y = (1.02)t</math>, <math>y = (0.97)t</math>, <math>y = (1.01)12t</math>, <math>y = (1.2)t/10</math>, and classify them as representing exponential growth or decay.</i></p>	<p>I can analyze a quadratic or exponential function by changing the format of a function to reveal particular attributes of its graph. This means:</p> <ul style="list-style-type: none"> <li>• I can factor to find the zeros of a quadratic function.</li> <li>• I can complete the square to show extreme values and symmetry.</li> </ul> <p>I can interpret important points on a quadratic graph in terms of a context.</p>
<p>F.IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). <i>For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.</i></p>	<p>I can compare properties of two functions represented differently (graphs, tables, equations, verbal descriptions) and draw conclusions based on those comparisons.</p>
<p><b>CCS Standards: Building Functions</b></p>	<p><b>Long-Term Target(s)</b></p>
<p>F.BF.1. Write a function that describes a relationship between two quantities.</p> <p>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.</p> <p>b. Combine standard function types using arithmetic operations.</p> <p><i>For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</i></p>	<p>I can describe a real-world context using a quadratic model.</p>
<p>F.BF.3. Identify the effect on the graph of replacing <math>f(x)</math> by <math>f(x) + k</math>, <math>k f(x)</math>, <math>f(kx)</math>, and <math>f(x + k)</math> for specific values of <math>k</math> (both positive and negative); find the value of <math>k</math> given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i></p>	<p>I can describe how a quadratic function can be transformed using a constant, <math>k</math>. This means:</p> <ul style="list-style-type: none"> <li>• I can experiment with different transformational constants and construct an argument about their effect on a quadratic functions using technology.</li> <li>• I can determine the transformational constant from graph of a quadratic (shifts and stretches, both vertical and horizontal).</li> </ul>

<p>F.BF.4. Find inverse functions.  a. Solve an equation of the form <math>f(x) = c</math> for a simple function <math>f</math> that has an inverse and write an expression for the inverse. <i>For example, <math>f(x) = 2x^3</math> or <math>f(x) = (x+1)/(x-1)</math> for <math>x \neq 1</math>.</i></p>	<p>I can determine the inverse of a linear function.</p>
<p><b>CCS Standards: Linear, Quadratic, &amp; Exponential Models</b></p>	<p><b>Long-Term Target(s)</b></p>
<p>F.LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</p>	<p>I can use tables and graphs to compare linear and exponential growth with quadratic growth.</p>